647510 **BSc6MatC601x** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
March/April-2024 (NEW COURSE)
Graph Theory and Complex Analysis-2 (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) ATTEMPT ANY TWO: (10)

(1) State and prove the handshaking lemma.

(2) Define: Euler Graph
Prove that a connected graph G is an Euler graph ***if and only if*** all vertices of G are of even degree.

(3) Define: Tree
Prove that a tree T with $\boldsymbol{n}$ vertices has $(\boldsymbol{n-1})$ edges.

Que-1(B) ATTEMPT ANY ONE : (04)

(1) What will be the smallest integer $\boldsymbol{n}$ for which the complete graph $\boldsymbol{K_n}$ has atleast 123 edges?

(2) Explain graph operations **UNION** and **DELETION** by taking suitable graphs.

Que-2(A) ATTEMPT ANY TWO: (10)

(1) Define: Planar Graph
Prove that $\boldsymbol{K_{3,3}}$, the regular graph with 6 vertices and 9 edges is non-planar.

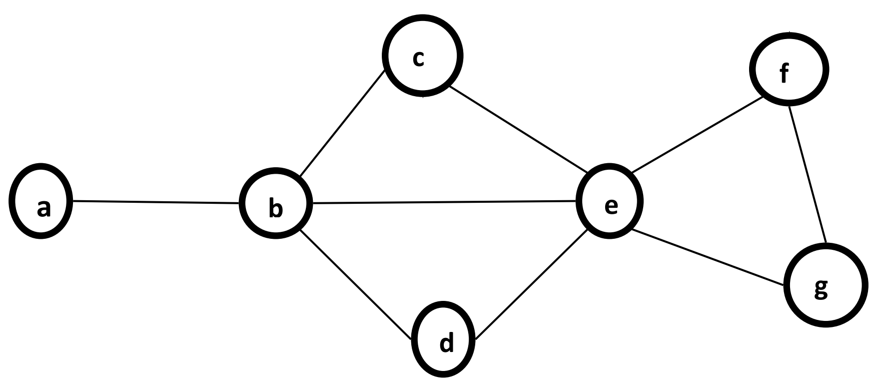
(2) State and prove the Euler's formula for connected planar graphs.

(3) For any graph G, prove that the set W_G of all vectors associated with subgraphs of G including the zero vector is a vector space over
$GF(2) = the\ Galois\ field\ of\ modulo\ 2.$

Que-2(B) ATTEMPT ANY ONE : (04)

(1) Write a short note on Geometric Dual of a graph.

(2) By applying the laws of Boolean Algebra, find all maximally independent sets of the following graph.



Que-3(A) ATTEMPT ANY TWO: (10)

(1) Define: Singular bilinear mapping.
Let $\boldsymbol{w_1, w_2, w_3}$ be the images of $\boldsymbol{z_1, z_2, z_3}$ respectively of $\boldsymbol{Z}$ −plane in $\boldsymbol{W}$ −plane under a bilinear transformation
$\boldsymbol{w = f(z) = \frac{az+b}{cz+d}}$, where $\boldsymbol{ad - bc \neq 0}$, then obtain the singular bilinear mapping.

(2) Prove that the Mobious Transformation maps circles or straight lines into circles or straight lines.

(3) Prove that the mapping $w = z^2$ is a Conformal Mapping.

Que-3(B) ATTEMPT ANY ONE : (04)

(1) Discuss the images of (a) line parallel to X-axis in Z-plane and (b) line parallel to Y-axis in Z-plane, under transformation $w = sin(z)$.

(2) Prove that the transformation $w = z^2 + 2z$ maps the unit circle $|z| = 1$ of the Z −plane into a cardioid in the W −plane.

Que-4(A) ATTEMPT ANY TWO: (10)

(1) If the complex function $f(z)$ is analytic everywhere inside and on circle C_0 having radius r_0 and centered at z_0 and z is any point inside the circle C_0, then prove that,

$$f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z - z_0) + \frac{f''(z_0)}{2!}(z - z_0)^2 + \frac{f'''(z_0)}{3!}(z - z_0)^3 + \cdots.$$

(2) State and prove Laurent's infinite series for an analytic function $f(z)$.

(3) Let $a \in (-1, 1)$ then derive Laurent's series representation

$\frac{a}{z-a} = \sum_{n=1}^{\infty} \frac{a^n}{z^n}$, $(|a| < |z| < \infty)$ and deduce the following from it.

$\sum_{n=1}^{\infty} a^n cosn\theta = \frac{acos\theta - a^2}{1-2acos\theta+a^2}$ <u>and</u> $\sum_{n=1}^{\infty} a^n sinn\theta = \frac{asin\theta}{1-2acos\theta+a^2}$

Que-4(B) ATTEMPT ANY ONE : (04)

(1) Prove that $\frac{sinhz}{z^2} = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{z^{2n-1}}{(2n+1)!}$.

(2) Find the Maclaurin series expansion of $f(z) = \frac{z}{z^4+9}$.

Que-5(A) ATTEMPT ANY TWO: (10)

(1) Define: (i) Simple pole (ii) Pole of order m

If z_0 is the pole of order $m(m \geq 2)$ of a complex function $f(z)$, then prove that

$Res(f(z), z_0) = \frac{\varphi^{m-1}(z_0)}{(m-1)!}$, $where\ \varphi(z) = (z - z_0)^m f(z)$.

(2) Prove: $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^3} = \frac{3\pi}{8}$

(3) Evaluate: $\int_C \frac{3z^2+2}{(z-1)(z^2+9)} dz$, $C: |z - 2| = 2$.

Que-5(B) ATTEMPT ANY ONE: (04)

(1) Evaluate: $\int_0^{\infty} \frac{cosax}{x^2+1} dx, a > 0$.

(2) Evaluate: $\int_0^{2\pi} \frac{d\theta}{(5+4sin\theta)}$

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